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SUMMARY OF RESEARCH

for

An Enhanced Multi-objective Optimization Technique for Comprehensive Aerospace Design

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Submitted by

**Principal Investigators: Aditi Chattopadhyay
John N. Rajadas**

**Arizona State University
Department of Mechanical and Aerospace Engineering
Tempe, AZ 85284**

ABSTRACT

An enhanced multiobjective formulation technique, capable of emphasizing specific objective functions during the optimization process, has been demonstrated on a complex multidisciplinary design application. The Kreisselmeier-Steinhauser (K-S) function approach, which has been used successfully in a variety of multiobjective optimization problems, has been modified using weight factors which enables the designer to emphasize specific design objectives during the optimization process. The technique has been implemented in two distinctively different problems. The first is a classical three bar truss problem and the second is a high-speed aircraft (a doubly swept wing-body configuration) application in which the multiobjective optimization procedure simultaneously minimizes the sonic boom and the drag-to-lift ratio (C_D/C_L) of the aircraft while maintaining the lift coefficient within prescribed limits. The results are compared with those of an equally weighted K-S multiobjective optimization. Results demonstrate the effectiveness of the enhanced multiobjective optimization procedure.

Introduction

Design of modern day aircraft is a multidisciplinary process involving the integration of several disciplines such as aerodynamics, structures, dynamics, and propulsion. In such a complex process, optimization techniques are valuable tools that enable the designer to choose a design point for the given aircraft configuration. These optimization techniques should be able to take into account the different disciplines associated with the aircraft design simultaneously. This can be a difficult task because desired performance criteria in the different disciplines involved in the design process often lead to conflicting requirements on vehicle configurations. One such optimization technique is the Kreisselmeier-Steinhauser (K-S) function approach [1] which has been well known in the mathematical programming community for a long time. Detailed discussion on the origin of this approach as well as the complex mathematical analysis leading up to the version of the technique used in the present work is beyond the scope of this paper since the current emphasis has been on the computational implementation of the technique for practical aerospace design applications. The K-S technique is a multiobjective optimization technique that combines all the objective functions and the constraints to form a single unconstrained composite function to be minimized. An appropriate unconstrained solver is then used to locate the minimum of the composite function. Any application where there are more than one design criteria to optimize is a candidate for this method. The K-S technique has already been shown to be effective in various complex multiobjective applications such as Tilt-Rotor design, High-Speed Civil Transport (HSCT) design, HSCT wing design, sonic boom minimization in HSCT, etc. [2-4].

An inherent characteristic of the K-S method is that all the objective functions or design criteria are equally weighted, which helps to eliminate problems associated with incorrect user input in setting up the optimization problems. However, in a multidisciplinary application, it would be advantageous to have a method where a designer could emphasize specific design criteria relative to the others. In

the present work, a technique has been formulated to allow a designer to have this capability while using the K-S method. The approach has been to modify the K-S functions using weight factors (unlike the usual way of equal weights on all the objective functions), thus enabling increased emphasis on specific objectives during the optimization process. The modified K-S function technique is referred to as Enhanced K-S technique in this paper. It must be reiterated here that the K-S function formulation has been chosen due to its ability to address multiple design objectives simultaneously in the design optimization process and the primary aim of the present work has been to enhance the technique and demonstrate it. The primary focus thus is the computational implementation and demonstration of the enhanced technique for practical design applications.

In the present work, the enhanced K-S multiobjective formulation technique has been applied to both a classical three bar truss problem and a HSCT sonic boom minimization problem. The three bar truss problem has been chosen to demonstrate the effectiveness of the method by comparing it to a known optimization problem. The use of the technique on the HSCT problem shows the effectiveness of the enhanced K-S method on a complex modern day aerospace application. The HSCT problem has competing design criteria that must be optimized. One such case is the apparent conflict between the design requirements for improved aerodynamic performance and better sonic boom characteristics of the airframe. For example, minimum lift to drag ratio (C_D/C_L) requires a slender forebody whereas minimum sonic boom designs usually have blunt forebodies. The following sections briefly outline the enhanced K-S function technique that has been implemented in the present work and the two problems used to demonstrate the procedure. More detailed information about the problems and the K-S approach can be found in the cited references.

Multiobjective Optimization

A general multiobjective optimization problem is,

$$\begin{aligned}
 &\text{Minimize/Maximize} && F_i(\Phi) && i = 1, 2, \dots, NF && \text{(objective functions)} \\
 &\text{subject to} && g_j(\Phi) \leq 0 && j = 1, 2, \dots, NC && \text{(inequality constraints)} \\
 &&& \Phi_L \leq \Phi \leq \Phi_U && && \text{(side constraints)}
 \end{aligned}$$

where Φ is the design variable vector, $F_i(\Phi)$ is the vector of objective functions, $g_j(\Phi)$ is the vector of constraints, NF is the number of objective functions to be optimized, and NC is the number of constraints imposed on the design optimization. The subscripts L and U denote lower and upper bounds, respectively, on the design variable vector. This is the general format of all the optimization problems addressed in the present work and the multiobjective method chosen to address this type of problem here is the K-S function approach [1].

Kreisselmeier-Steinhauser (K-S) Function Technique

In the K-S function approach [1], the original objective functions are transformed into reduced or normalized objective functions [2]. Depending on whether these functions are to be minimized or maximized, they can be expressed as,

$$\hat{f}_i(\Phi) = \frac{F_i(\Phi)}{F_{i_0}} - 1.0 - g_{\max} \leq 0, \quad i = 1, \dots, \text{NF (minimization)}$$

$$\hat{f}_i(\Phi) = 1.0 - \frac{F_i(\Phi)}{F_{i_0}} - g_{\max} \leq 0, \quad i = 1, \dots, \text{NF (maximization)}$$

where F_{i_0} represents the value of the original objective function corresponding to the current reference design variable vector for a given optimization cycle, and F_i is the value of the original objective function which is dependent on the design variable vector. F_{i_0} is constant during a given optimization cycle. g_{\max} is the largest value of the original constraint vector at the current reference point and is held constant during each iteration (cycle). Since the reduced objective functions are analogous to the original constraints, a new constraint vector $\mathbf{f}_m(\Phi)$, $m = 1, 2, \dots, M$, where $M = \text{NC} + \text{NF}$, is introduced. The first NC elements of \mathbf{f}_m are the original constraints and the next NF elements are the reduced objective functions. The original constrained optimization problem with multiple objective functions is thus transformed into a single-objective, unconstrained minimization problem. Now the problem is to minimize the K-S function, $F_{\text{KS}}(\Phi)$, defined as

$$F_{\text{KS}}(\Phi) = f_{\max} + \frac{1}{\rho} \log_e \sum_{m=1}^M e^{\rho(f_m(\Phi) - f_{\max})}$$

where f_{\max} is the largest constraint corresponding to the new constraint vector $\mathbf{f}_m(\Phi)$ (in general not equal to g_{\max}). When the original constraints are satisfied during optimization, the constraints due to the reduced objective functions are violated. Initially, in an infeasible design space, where the original constraints are violated, the constraints due to the reduced objective functions are satisfied (i.e., g_{\max} is negative). The optimizer attempts to satisfy the violated constraints, thus optimizing the original objective functions (F_i).

The parameter ρ , which is analogous to the draw-down factor of penalty function formulation, controls the distance from the surface of the K-S envelope to the surface of the maximum constraint function. When ρ is large, the K-S function will closely follow the surface of the largest constraint

function and when ρ is small, the K-S function will include contributions from all constraints. The new unconstrained minimization problem can be solved by using a variety of techniques. In the present work, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [5] has been used. This algorithm approximates the inverse of the Hessian of the composite objective function using a rank-two update and guarantees both symmetry and positive definite characteristics of the updated inverse Hessian matrix. The K-S formulation coupled with the BFGS algorithm has been successfully applied to a variety of aerospace design applications [2-4].

The role played by ρ in the implementation of the K-S function formulation is illustrated in Figs. 1-2 for an optimization problem with two objective functions to be minimized and one constraint. The objective functions and the constraint are functions of a single design variable, ϕ (an initial design point of $\phi_0 = 0.5$ is used). Initially, the constraint is satisfied and, therefore, g_{\max} is negative. The original constraint and the two additional constraints from the two reduced objective functions are shown in Fig. 2 along with the K-S function envelopes for two different values of ρ . For $\rho=1$, the K-S function includes equal contributions from all the three constraints. For the larger value of $\rho=3$, the K-S function gets a stronger contribution from the largest constraint and weaker contributions from the other two. Thus large values of ρ “draw down” the K-S function closer to the largest constraint. The value of ρ may change from cycle to cycle in the optimization process. In a typical application, it is progressively increased so that as the optimization proceeds the K-S function more closely represents only the largest constraint.

Enhanced K-S Function Technique

As mentioned above, the main focus of the present work has been to enhance the K-S approach in such a way as to enable the ability to emphasize specific objective functions during the design optimization process. This gives the designer the option and ability to focus the design on specific areas of concern (especially in component designs). Towards this end, the reduced objective functions have been modified to allow relative weighting of specific design criteria. This is achieved by incorporating a vector of weight factors β_i ($i = 1, 2, \dots, NF$) in the K-S envelope [6] as shown below.

$$\hat{f}_i(\Phi) = \frac{\beta_i F_i(\Phi)}{F_{i_0}} - \beta_i - g_{\max} \quad i = 1, \dots, NF$$

The total number of weight factors is equal to the number of objective functions. The relative magnitudes of β_i will help to emphasize specific objective functions in the overall optimization

process. The weight factors (β_i) are positive numbers the numerical values of which are dictated by the specific application. The original unweighted K-S formulation is recovered if $\beta_i = 1$.

In the present work, two different methods of weighting have been investigated. The first one (Type A) involves assigning positive integer values larger than unity as the weight factor for the objective function to be emphasized, while assigning a weight factor of unity to all other objective functions. In the second approach (Type B), unity is assigned to be the weight factor for the emphasized objective function, while assigning a positive value smaller than unity to the remaining objective functions. In the sections below, the following definitions are employed to identify the two methods described above.

Type A : Weight factors for emphasized objective functions are positive integer value larger than unity, while all other weight factors are assigned a value of unity.

Type B : Weight factor for emphasized objective function is unity, while all other weight factors take on values less than unity.

Results and Discussion

In multidisciplinary optimization problems involving complex analyses (such as the HSCT problem addressed here), competing design attributes are almost always present. This usually leads to the requirement that multiple design objectives be included simultaneously in the optimization process and a procedure capable of addressing multiple objective functions and constraints be used. All the problems chosen in the present work to demonstrate the enhanced optimization procedure involve objective functions that impose conflicting design requirements. Two different problems have been chosen to demonstrate the enhanced K-S formulation. The first is a classical three bar truss problem with two objective functions and six constraints and the second is the HSCT airframe design problem for improved aerodynamic performance and sonic boom characteristics which involves three objective functions, three constraints, and six design variables. The number of design variables and constraints have been kept to a minimum here since the primary aim of the work is to demonstrate the enhanced procedure as well the computational implementation of the procedure in a complex design problem.

Three-Bar Truss Problem

The first application of the enhanced K-S function is a classical three bar truss problem [1]. A modified version of the three bar truss problem used to demonstrate the original K-S formulation [1] has been chosen. A schematic of the problem is shown in Figure 3. The two outside bars of the truss are made of steel, and the middle bar is made of titanium. Two loads are applied as shown. The material properties and costs of the truss are also shown on the figure. The objective is to minimize both the weight and the cost of the truss. The optimization problem is as follows.

Minimize

Weight of the 3-Bar truss, W

Cost of the 3-Bar truss, C

subject to

$$S_{ct} \leq S_i \leq S_{yt}, \quad i = 1 - 3$$

There are two objective functions, six constraints, and two design variables in the optimization problem. The design variables are the cross sectional areas of the truss members, A_1 and A_2 , (Figure 3) which are required to be greater than 0.001 square inches. There are three constraints on the tensile loads and three on the compressive loads. Since titanium is lighter than steel, the minimum weight design is expected to have a larger titanium center member and smaller steel outer members. The minimum cost design would have a smaller titanium center member and larger steel outer members since steel is cheaper than titanium. These conflicting design criteria make this problem a good candidate for demonstrating the enhanced K-S technique.

Preliminary optimization was carried out for the following three cases (Figs. 4-5):

- a) Single objective, weight minimization ("weight only")
- b) Single objective, cost minimization ("cost only")
- c) Multiobjective, unweighted optimization ("(1,1)").

These are used as reference cases for comparison with the results of the enhanced optimization. The expected trends of weight and cost variations are seen. Also, Fig. 5 indicates that the minimum cost criteria is the critical one in this optimization problem. The results obtained by using the enhanced multiobjective optimization process on the 3-Bar Truss Problem are presented in Figures 6-9. In the figures, the weight factor set (5,1) means that the first objective function (weight) has received a weight factor of 5 while the second (cost) has a weight of 1 leading to increased emphasis on the first objective function during the optimization process. The unweighted K-S formulation is recovered when a weight factor combination of (1,1) is applied.

Figures 6-7 show the results of weighting the first objective function (weight) using Type A weight factors. Weight factors of 2, 5, 10, and 100 relative to the cost have been chosen to emphasize the minimum weight criterion here. The results show that the enhanced K-S approach is effective in emphasizing a specific objective in the multiobjective optimization problem. - For example, when the weight of the truss is emphasized (Fig. 6), the decrease in weight with increasing emphasis (weight factor varies from 1 to 10) is seen. While using Type A weight factors, it is apparent that the magnitude of the weight factor may be bounded for the specific problem at hand. This is seen by comparing the results of weight factor sets (10,1) and (100,1) which implies that in the multiobjective optimization process, all the objectives impact the form of the optimum design even if one of them is being emphasized more in comparison with the rest. The effect of the weighting on the design variables is shown in Table 1. Figures 8-9 show the results of the Type B weight factors being used

to emphasize the weight. From the table and the figures, it is apparent that the Type B weight factors did have the expected effect, however it took a fairly small weight factor to achieve it.

The numerical value of the weight factor(s) depends on the specific application being addressed. User input and experience thus are important factors in the optimization process. The change in the objective function with increasing weight factor is nonlinear. That is, a very large value of the weight factor does not always lead to the lowest value of the objective function. The reason for that may be that a very large weight factor may have the effect of forcing the design into the infeasible domain leading to infeasible designs. For the present problem, it appears that the Type B weight factors are preferable because they can achieve the desired effect (with correct user input) in a more stable (less number of forays into the infeasible design space) manner than the Type A factors.

HSCT Sonic Boom Minimization Problem

The second problem addressed in this work is that of a High Speed Civil Transport (HSCT) design for minimum sonic boom and improved aerodynamic performance [4]. Figure 10 illustrates a typical sonic boom (pressure) signature produced by a supersonic aircraft (wing-body configuration) at a distance from the aircraft. The two positive pressure peaks are the sonic boom levels that must be minimized. The first peak (Δp_{\max_1}) is caused by the bow shock associated the nose of the aircraft. The second peak (Δp_{\max_2}) is caused primarily by the leading edge of the wing. From an aerodynamics perspective, a primary objective is to minimize the lift-to-drag ratio (C_D/C_L). Thus, the three objective functions to be minimized for the HSCT optimization problem are the two pressure peaks and the C_D/C_L ratio. This must be accomplished while keeping the lift produced by the aircraft at a desired level, which is done by imposing upper and lower limits on the C_L ($C_{L_{\max}}$ & $C_{L_{\min}}$). A constraint has also been placed on the wing trailing edge angle to ensure computational stability. The mathematical formulation of the problem is as follows.

Minimize

Drag to Lift Ratio, C_D/C_L

Over pressure Peaks, Δp_{\max_1} , Δp_{\max_2}

subject to

$$C_{L_{\min}} \leq C_L \leq C_{L_{\max}}$$

Lift Constraint

$$\lambda_{te} \leq \frac{\pi}{2} \text{ rad}$$

Wing Trailing Edge Constraint

$$\Phi_L \leq \Phi \leq \Phi_U$$

Side Constraints on Design Variables

In an effort to keep the computational effort low, preliminary computations for optimum forebody geometry for minimum first pressure peak and minimum C_D/C_L were carried out. The forebody design (the nose length and the maximum radius of the forebody) is frozen at the level prescribed by

this optimum design for the subsequent computations involving the wing. Hence, the design variables for the configuration used for the present work are all associated with the wing and its location along the length of the aircraft (Figure 11). As a result, only the second pressure peak (Δp_{\max_2}) and the C_D/C_L ratios are the relevant objective functions to be addressed here.

The six design variables chosen for the present study are: the wing root chord (c_0), the two leading edge sweep angles (λ_1 & λ_2), the tip chord (c_t), the break length (x_b), and the wing starting location (x_w). Upper and lower bounds (side constraints) are imposed on these geometric variables during the optimization process. While the first pressure peak remains an objective function, only the second pressure peak and the C_D/C_L ratio have been weighted as part of the enhanced K-S formulation to expedite the solution procedure. This was done with a view to keep the computational cost and turnaround time low since the aerodynamic analysis is carried out by a three dimensional Navier-Stokes solver. The inviscid flow field for the wing-body geometry has been evaluated using the flow solver UPS3D [7] that utilizes the three dimensional Parabolized Navier Stokes (PNS) equations. An extrapolation technique [8], has been used to obtain the sonic boom signatures from the flow field pressure data in the present work.

For the weighting factors of the enhanced K-S formulation, the order of the objective functions (F_i) is: C_D/C_L ($i = 1$) is first and then the first and second pressure peaks ($i = 2, 3$). Thus, a (5,1,1) weight factor set indicates that C_D/C_L is weighted by a factor of 5 relative to $(\Delta p_{\max})_1$ and $(\Delta p_{\max})_2$. In the results presented here, the subscript “ref” indicates the configuration before the optimization process begins. As mentioned before, only the first and third objective functions (C_D/C_L , Δp_{\max_2}) have been assigned weighting factors. The optimum results presented here correspond to those obtained at the end of 30 optimization cycles.

The sensitivity analysis for the HSCT application was carried out using a finite difference approach where the design variables are perturbed by a prescribed amount and the CFD solver is used repeatedly on the “perturbed” configurations [6]. The results from the perturbed and unperturbed configurations are then used for calculating the sensitivities. This approach has its inherent accuracy problems in addition to the large computational time associated with the three dimensional CFD solver. Also, the two-point exponential approximation technique [9] used to advance from cycle to cycle may give rise to deviations from a true design point. Such deviations and errors may sometimes be magnified if the problem being addressed (e.g. HSCT) is complex involving large analysis tools. The results of the present section should be viewed with these considerations as a backdrop.

Figures 12-13 show the effect of the weight factors on the objective functions. The optimum solutions (after 30 cycles) obtained for unweighted ((1,1,1)), C_D/C_L -emphasized ((10,1,1) and (1,0.1,0.1)) and $(\Delta p_{\max})_2$ -emphasized ((1,1,10) and (0.1,0.1,1)) are compared along with the

reference values of the objective functions of interest (C_D/C_L and $(\Delta p_{\max})_2$). Tables 2-3 also contain the minimum values achieved for C_D/C_L and $(\Delta p_{\max})_2$ for each weight factor set. Also shown in the tables are the corresponding design variables for these cases and the corresponding number of optimization cycles.

Figure 12 and Table 2 present the results for the case where minimum C_D/C_L is the primary focus and the results show the effectiveness of weight factors in emphasizing specific design objective(s) in a multiobjective design optimization problem. The weight factor sets that emphasized C_D/C_L achieve a lower C_D/C_L than the unweighted case, and the weight factor sets that emphasize $(\Delta p_{\max})_2$ have a larger minimum C_D/C_L than the unweighted case. Figure 13 and Table 3 show that all the weight factor sets achieved lower values for $(\Delta p_{\max})_2$ than the unweighted case. The lowest minimum was found with the Type A weight factor set that was designed to emphasize C_D/C_L . It must be noted that this set did in fact achieve the results desired for emphasizing C_D/C_L . The main reason for the occurrence of the lower value of $(\Delta p_{\max})_2$ here could be that the optimization formulation tends to favor the objective function with the lowest value ($(\Delta p_{\max})_2$ here) [6]. This can also be seen from the data presented in Tables 2-3. Also, there is another important observation to be made here. Even though the weighting of a single objective function in relation to the others might seem to imply a “single objective” optimization problem, the truly multiobjective nature of the complex problem is evident in these results. The pressure signatures associated with the minimum $(\Delta p_{\max})_2$ cases are shown in Figure 14. The variations in the second pressure peak can be seen.

One of the key issues to be addressed in the enhanced K-S function procedure is the proper choice of weight factors. In the present work, two types (Type A and Type B) have been examined. Type B weight factors consistently were more robust than Type A weight factors in arriving at the optimum values for the specified objective functions in this problems as well as the previous (three-bar truss) one. Based on the results obtained in this study, Type B weight factors are recommended to be used with the developed procedure. However, a more detailed study of the appropriate form (such as a normalized set of weight factors) of the weight factors and their effect on the optimization process in general is necessary.

Conclusions

The primary goal of the present work has been to enhance the multiobjective K-S function based optimization procedure by adding the capability of selectively emphasizing specific objective functions. This has been achieved by incorporating weight factors for the objective functions. These weight factors allow a designer to take advantage of the characteristics of the original K-S formulation while retaining the ability to emphasize selected area(s) of the design. The effectiveness of the weight

factors has been demonstrated on two very different problems. The calculations show that the enhanced multiobjective formulation is suitable for a wide spectrum of design (aerospace and other) problems, especially in multidisciplinary designs where conflicting design requirements may exist. The associated problem of the appropriate form of weight factors (relative magnitudes) to be used with the optimization procedure also has been addressed. It is concluded that among the various forms employed in the present work, the Type B weight factors, where the maximum value of β is 1.0, will lead to a robust optimization process. Also, the suitability of the procedure (enhanced K-S function technique) in the modern design environment has been demonstrated on the complex multidisciplinary, multiobjective design optimization problem associated with the HSCT design problem. It must be reiterated here that the K-S function formulation has been chosen due to its ability to address multiple design objectives simultaneously in the design optimization process and the primary aim of the present work has been to enhance the technique and demonstrate it for practical design applications.

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Table 1. Effect of Weight Factors on Design Variables

Weight Factor Sets	A1 (in ²)	A2 (in ²)
(1,1)	0.555	0.001
(2,1)	0.505	0.001
(5,1)	0.477	0.001
(10,1)	0.455	0.001
(100,1)	0.442	0.318
(1,0.5)	0.555	0.001
(1,0.2)	0.555	0.001
(1,0.1)	0.555	0.001
(1,0.01)	0.453	0.347

Table 2. Minimum C_D/C_L for weight factor sets.

	ref	(1,1,1)	(10,1,1)	(1,0.1,0.1)	(1,1,10)	(0.1,0.1,1)
λ_1 (deg)	70.46	72.86	74.50	72.84	72.32	72.51
λ_2 (deg)	52.42	51.43	52.87	50.50	50.81	50.58
c_o (m)	7.81	8.29	8.67	8.30	7.96	7.86
c_i (m)	1.5776	1.3510	1.2666	1.2632	1.3165	1.2400
x_b (m)	11.99	12.51	13.00	12.39	12.36	12.30
x_w (m)	7.80	7.56	7.71	7.27	7.62	7.66
C_D/C_L	0.11196	0.11049	0.11028	0.11016	0.11075	0.11077
		(-1.3%)	(-1.5%)	(-1.6%)	(-1.1%)	(-1.1%)
$(\Delta p_{\max})_2$	0.05206	0.04700	0.04336	0.04735	0.04680	0.04551
		(-9.7%)	(-17.2%)	(-9.0%)	(-10.1)	(-12.6)
cycle	-	29	28	27	18	27-

Table 3. Minimum $(\Delta p_{\max})_2$ for weight factor sets.

	ref	(1,1,1)	(10,1,1)	(1,0.1,0.1)	(1,1,10)	(0.1,0.1,1)
$\lambda_1(\text{deg})$	70.46	73.59	74.50	73.76	73.01	73.23
$\lambda_2(\text{deg})$	52.42	51.95	52.35	50.25	50.25	50.25
$c_o(\text{m})$	7.81	8.21	8.59	8.22	7.80	7.94
$c_i(\text{m})$	1.5776	1.3375	1.2794	1.2400	1.2400	1.2400
$x_b(\text{m})$	11.99	12.39	12.87	12.34	12.25	12.42
$x_w(\text{m})$	7.80	7.64	7.63	7.31	7.69	7.74
C_D/C_L	0.11196	0.11086	0.11035	0.11032	0.11100	0.11092
		(-1.0%)	(-1.4%)	(-1.5%)	(-0.9%)	(-0.9%)
$(\Delta p_{\max})_2$	0.05206	0.04442	0.04285	0.04428	0.04428	0.04391
		(-14.7%)	(-17.7%)	(-14.9%)	(-15.0)	(-15.7)
cycle	-	30	27	30	29	30

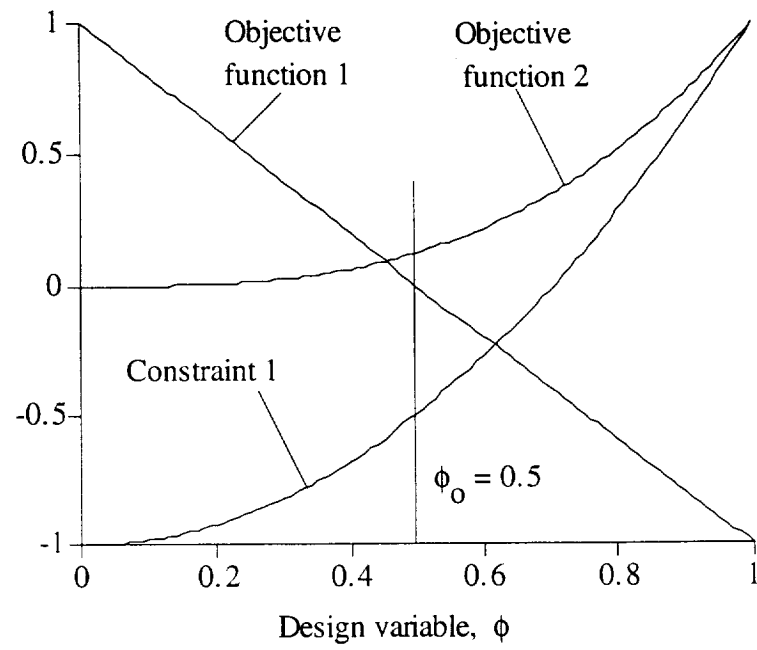


Figure 1. Original objective functions and constraints.

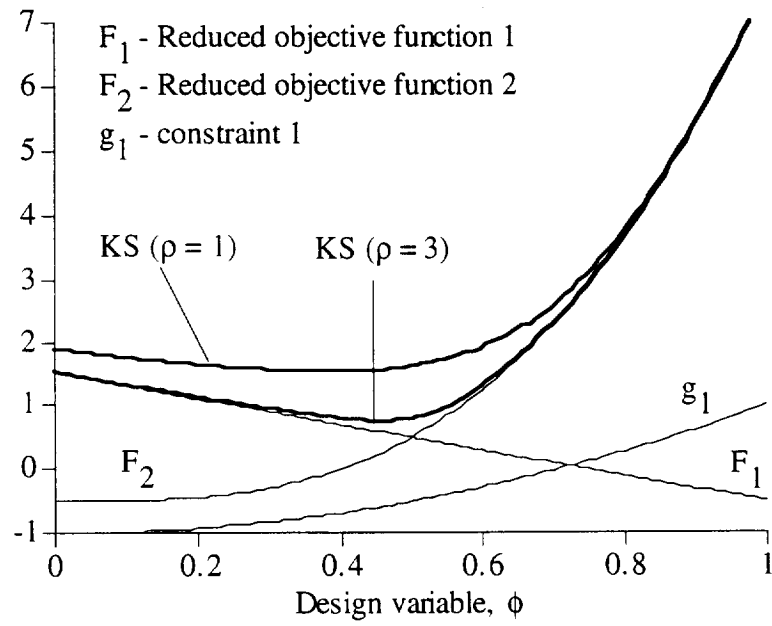
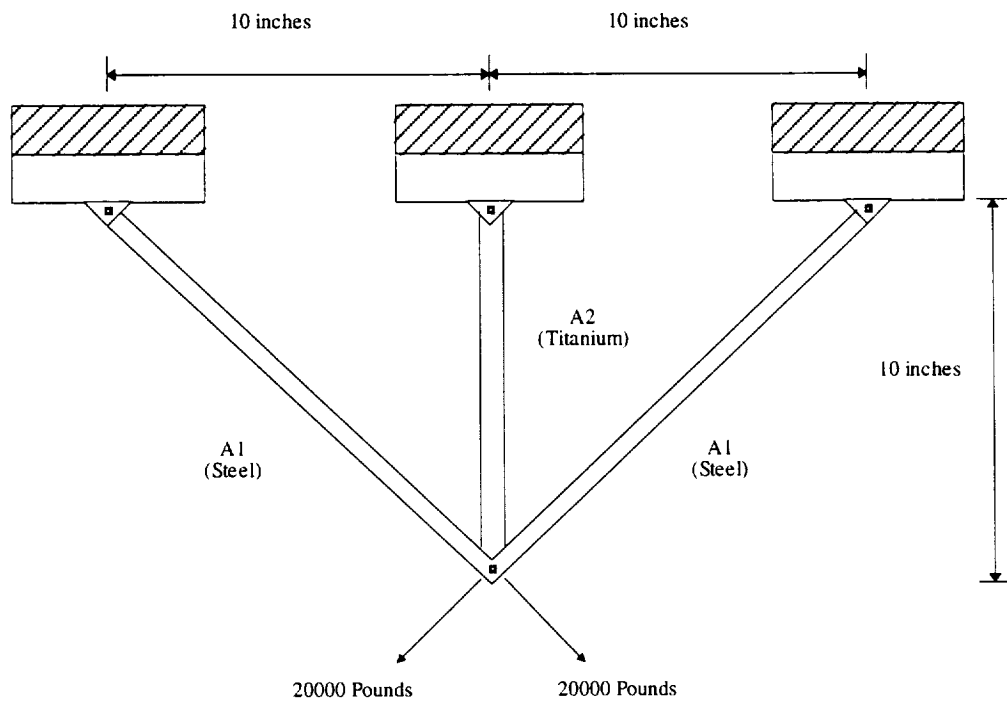


Figure 2. K-S function envelope.



Material Properties	Steel	Titanium
Young's Modulus(psi)	30,000,000	15,500,000
Density (lb/cu in)	0.282	0.160
Cost(\$/lb)	0.41	25.00
Tensile Yield Stress(psi)	36,000	110,000
Comp. Yield Stress(psi)	27,000	82,500

Figure 3. Three Bar Truss Example Problem with Material Properties.

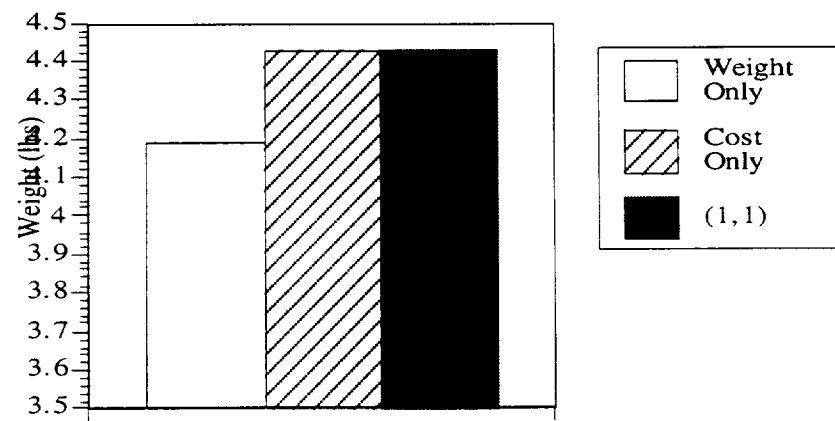


Figure 4. 3-Bar Truss: Weight.

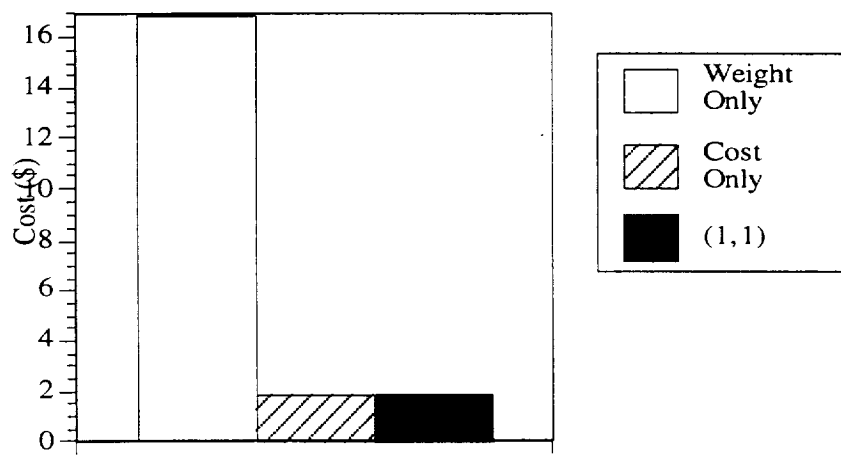


Figure 5. 3-Bar Truss : Cost.

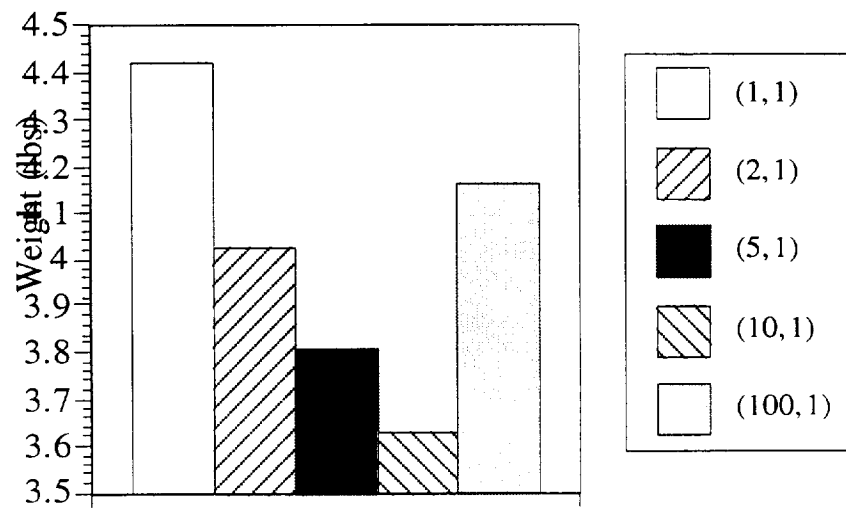


Figure 6. Weight for Type A Weight Factors.

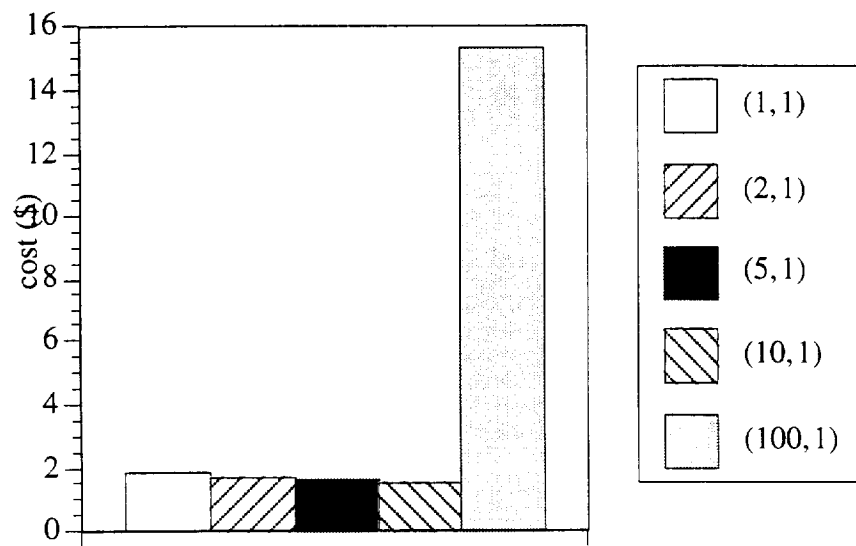


Figure 7. Cost for Type A Weight Factors.

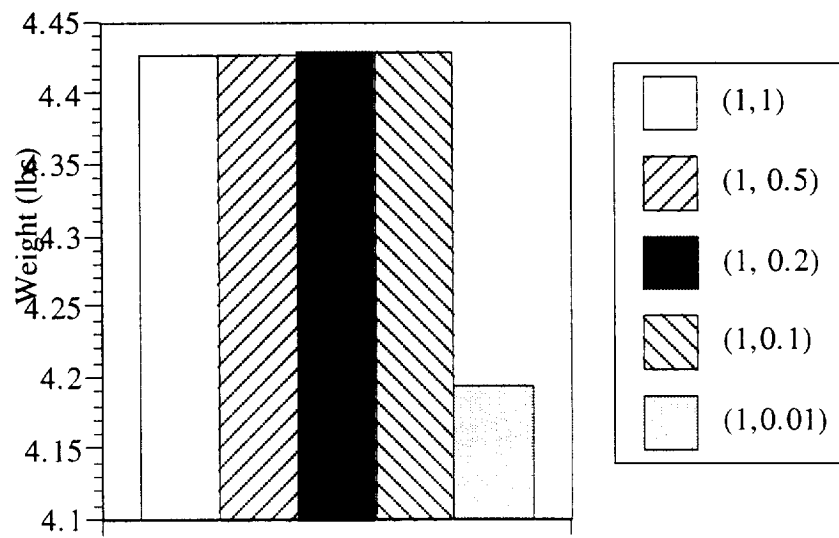


Figure 8. Weight for Type B Weight Factors.

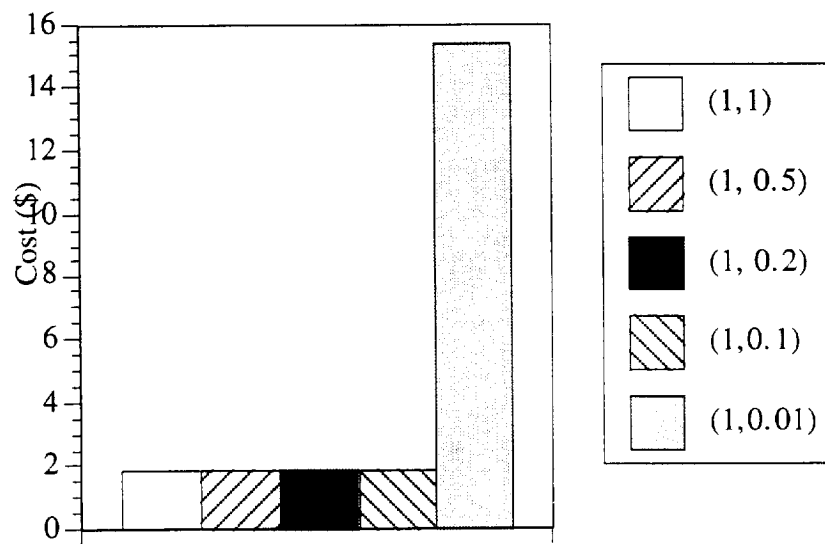


Figure 9. Cost for Type B Weight Factors.

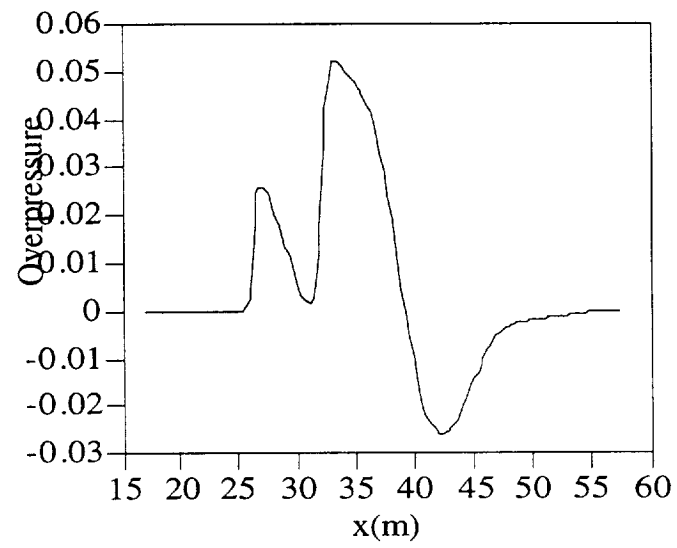


Figure 10. Sonic boom pressure signature of a supersonic wing-body configuration.

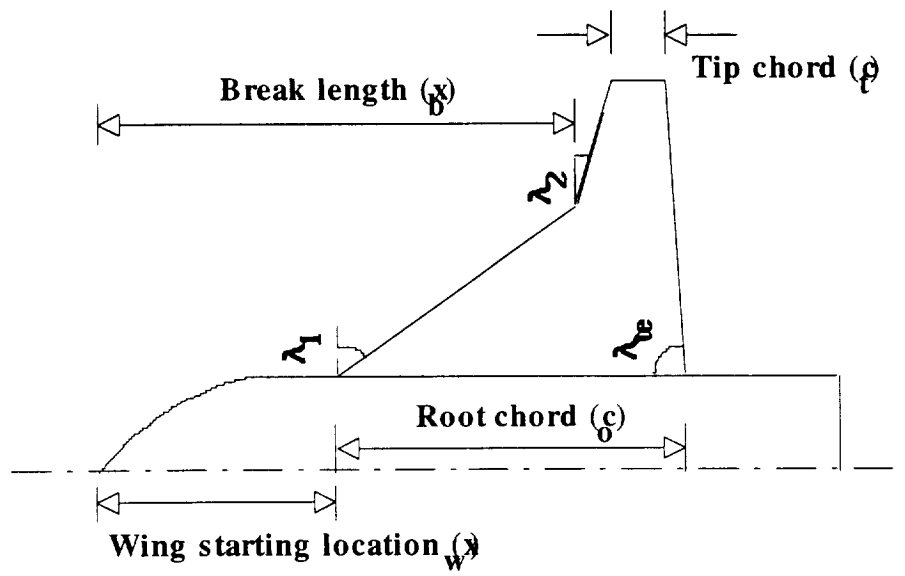


Figure 11. HSCT configuration and design variables.

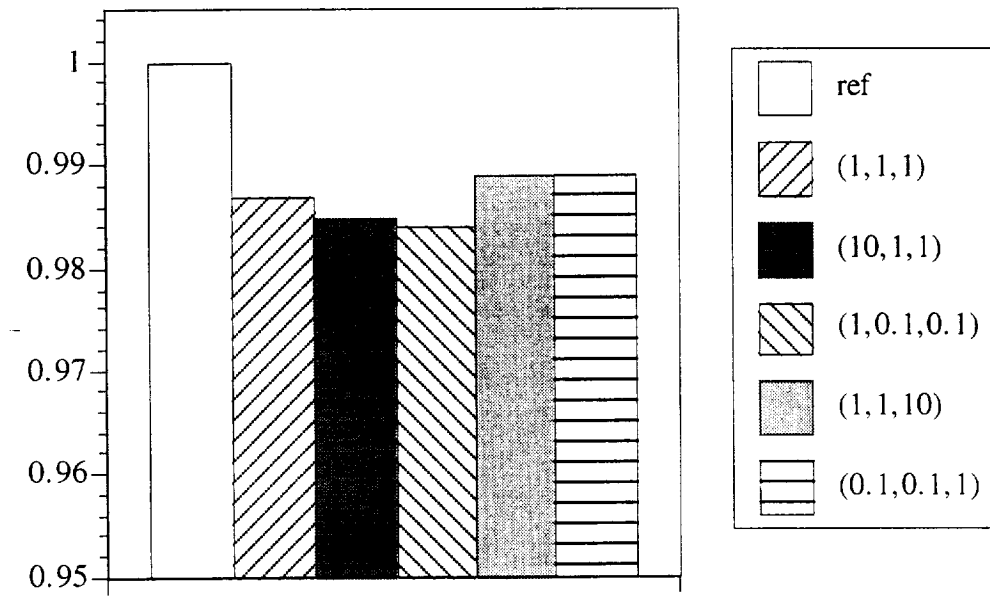


Figure 12. Comparative results of minimum C_D/C_L for each weight factor set.

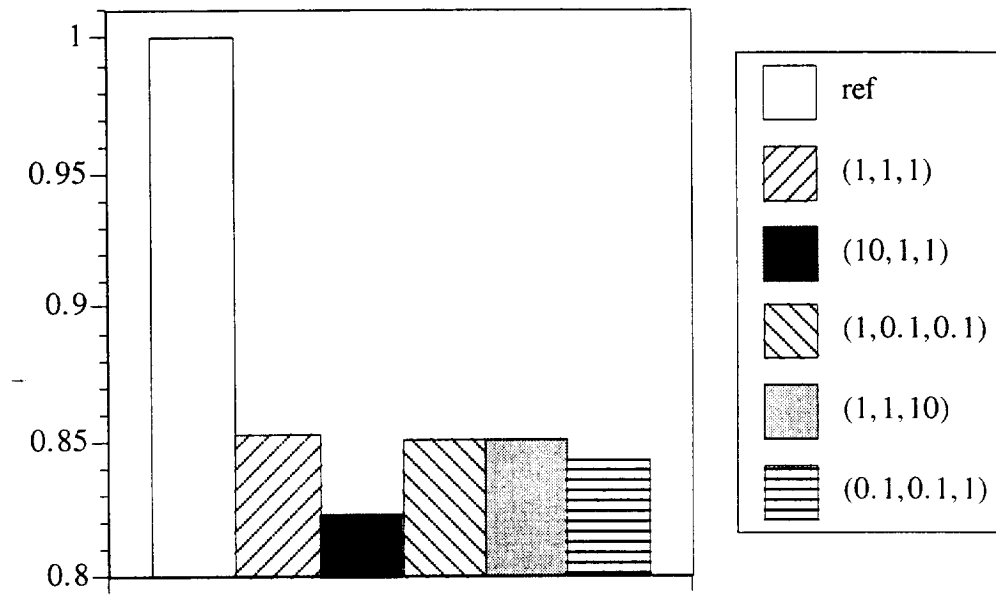


Figure 13. Comparative results of minimum $(\Delta p_{\max})_2$ for each weight factor set.

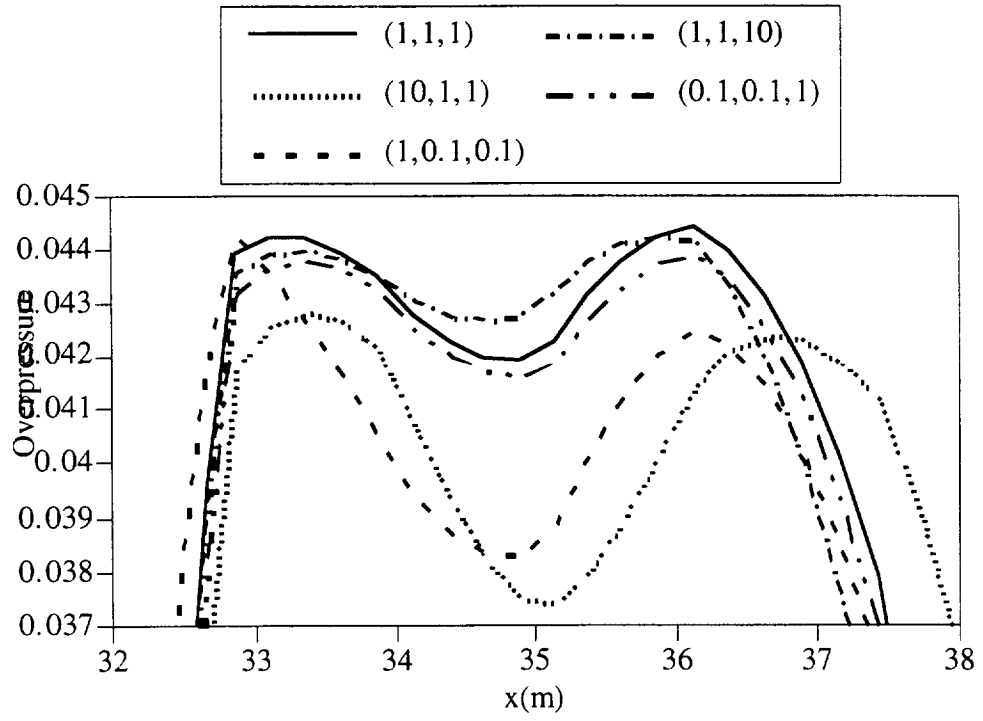


Figure 14. The second pressure peak for minimum $(\Delta p_{\max})_2$ cases.